An Assessment of Comparative Efficiency Measurement Techniques

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Europe Economics
FOREWORD

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The present paper has been prepared by Vasilis Sarafidis. Vasilis, following a year of full-time employment as an analyst at Europe Economics, is working part-time whilst studying for a PhD in econometrics at the University of Cambridge. The paper provides an assessment of different comparative efficiency measurement techniques and offers a guide to the reader on the use of these methods in practice.

It is hoped that this work will provide a relevant technical contribution to an important area of current debate.

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1 INTRODUCTION

1.1 The Role of Comparative Efficiency Analysis

Introductory economics textbooks often assume that producers are successful optimisers. However, casual observation suggests that although producers may indeed attempt to optimise, they do not always succeed, with the result that some producers appear to be consistently more efficient than others. Where there is competition, market forces may drive inefficient companies out of the market and prices fall as more efficient practices are adopted, to the benefit of consumers. For some industries competitive forces may not be sufficient to penalise firms that are inefficient. A regulator may seek to “imitate competition” by setting prices that reflect a best guess of the costs that an efficient firm would incur. One tool used to estimate the production or cost frontier is comparative efficiency analysis.

For example, in the UK, the economic regulators of the water and electricity sectors (Ofwat and Ofgem) have both used comparative efficiency analysis in order to determine the price controls on regional water suppliers and regional electricity distribution companies.1

The UK regulator for the telecommunications industry (Oftel) has examined the efficiency of BT relative to US Local Exchange Carriers as part of its effort to set the X factors in the RPI–X price cap formula used for retail and network price controls.2

The Dutch and Norwegian Electricity Regulators (DTe and NVE respectively) have both relied on benchmarking methods for electricity distribution companies to provide incentive mechanisms to cost savings.3

The incumbent fixed telecommunications operator in Denmark, TDC, has used comparative efficiency analysis in developing its top-down model to estimate interconnection charges and the costs of the local loop and co-location services.

Comparative efficiency analysis has also become important in the UK healthcare industry where growing public expectations about services of higher quality have not always been accompanied by proportional increases in the budget for health and therefore efficiency in the use of resources by all health authorities has become a major concern among policy makers.

Thus, comparative efficiency analysis is increasingly recognised as a useful tool for benchmarking and incentive regulation. It can help managers to identify and remedy underperformance, and regulators to encourage efficiency and ensure that consumers benefit from the resulting efficiency gains.

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Introduction

In particular, in the context of regulation, since companies are unlikely to all be equally efficient, corrections in the price controls for differences in efficiency can remove an element of injustice. Where the same future efficiency projections are assumed for all firms, efficient companies can suffer financially from having to meet average efficiency improvement targets, while inefficient companies would not be expected to catch up with best practice. Where differences in efficiency are significant — which they often are — measuring relative efficiency becomes an extremely important issue.

The purpose of this paper is two-fold: first, to identify some of the different techniques that can be used to measure relative efficiency, and second, to provide a high-level assessment of their merits as well as give a useful guide as to which technique is more appropriate in different circumstances.

1.2 Overview of Comparative Efficiency Measurement Techniques

There are two main approaches to estimating relative efficiency across firms: the parametric approach (or, more strictly, the statistical approach) and the non-parametric approach. The main difference between these two approaches is that the former specifies a particular functional form for the production or cost function while the latter does not. In fact, the degree of “parameterisation” of the production or cost function can have serious implications in comparative efficiency analysis, and can be considered to be responsible for the different advantages and disadvantages that each approach has.

The parametric approach relies on econometric techniques and includes simple regression analysis and Stochastic Frontier Analysis (SFA). Whilst simple regression analysis typically seeks to estimate a production or cost function, SFA is an extension of that methodology to estimate the “frontier” of a set of functions with different underlying levels of efficiency.

The non-parametric approaches use mathematical programming techniques, and the main non-parametric frontier analysis technique, known as Data Envelopment Analysis (DEA), can be seen as an extension of the simple technique of index numbers.

A possible taxonomy of efficiency techniques is illustrated in Figure 1.
1.3 Structure of this Paper

The paper is structured as follows:

- Section 2 describes simple regression analysis;
- Stochastic frontier analysis is illustrated in section 3;
- Section 4 describes data envelopment analysis.
- Section 5 concludes.

Recent developments such as the application of panel data analysis in the measurement of comparative efficiency as well as some advances in the DEA literature are discussed in the appendix.
2 SIMPLE REGRESSION ANALYSIS

2.1 Introduction

Simple regression analysis entails the use of the method of least squares for estimating – among other statistical relationships between variables – production or cost functions and thereby for measuring relative efficiency within a sample of comparators. Least squares is a method for fitting the “best” line to the sample and involves minimising the sum of the squared (vertical) deviations of actual observations from the fitted line.

Simple regression analysis has been used extensively by a number of regulators in UK such as Ofgem (to the exclusion of any other method), Ofwat and Oftel.

The use of regression analysis in estimating relative efficiency may be best explained by building up a simple example.

2.2 Simple Regression Analysis

The task is to assess the cost efficiency of companies in a particular sector. For simplicity we assume that each company incurs some aggregated costs in order to produce a number of outputs.

These outputs will vary from sector to sector and could be the number of access lines, local and main switched minutes and private circuits for the telecom sector, the number of properties and the amount of water delivered for the water sector and finally the number of customers, the number of units distributed and the number of transformers for the electricity sector. This list of outputs is by no means exhaustive and serves only for indicative purposes.

A simple method to measure relative efficiency would be to use the following two-step procedure. In the first step, a simple regression model would be formulated in order to try to identify the relationship that best fits the observed data. This relationship would show the amount of the total variation in cost that can be attributed to variations in the cost drivers (outputs).

In the second step, the residuals of the estimated regression (that is, the difference between actual cost and the cost predicted by the fitted line) would be treated as measures of inefficiency.

Box 1 – The Ordinary Least Squares model

The regression model can be written as follows:

\[ c_i = f(y_i; \beta) + u_i \]

where \( c_i \) represents total cost, \( y_i \) represents a vector of outputs, \( \beta \) is a vector of parameters to be estimated, \( f \) is the functional form that characterises the relationship between \( c \) and \( y \), and \( u_i \) is the estimate of inefficiency for firm \( i \).
This process is illustrated in figure 2, for the case of a log-linear model. According to the graph, firms C, E and F would be considered to be relatively efficient as they lie below the fitted regression line and firms A, B, and D relatively inefficient since they lie above this function. This method is usually termed “simple regression analysis”, or ordinary least squares (OLS) regression.

An extension of OLS is Corrected Ordinary Least Squares (COLS). This method shifts the estimated cost function downwards until all the residuals – in this case, the difference between actual and predicted cost – are positive (except for the company or companies found efficient, for which the residual is zero). COLS regressions share the weaknesses of OLS regressions, which are discussed below.

### 2.3 Strengths and Weaknesses

The main attraction of regression analysis is that it is computationally easy and straightforward; it part of any statistical package and most spreadsheet softwares.

An additional important advantage is that regression analysis – being a parametric method – provides pragmatic rules that can help in the formulation of a comprehensive strategy for deciding upon competing models of measuring relative efficiency.

Moreover, regression analysis can easily estimate the impact of environmental factors on the company’s efficiency. The term environment is used here to describe factors other than outputs that could influence the efficiency of the companies and are assumed to be outside the control of the management.
For example, in the telecommunications industry the most important environmental factors are of geographical nature. In particular, one would expect that operators that operate in denser but less dispersed (in terms of lines) areas would incur lower costs, all other things being equal.

The same reasoning applies in the water and gas sectors, where it is expected that cost “efficiency” depends on geographical characteristics such as terrain, climate and various population features.

As such environmental factors are outside the control of the management, it would be unfair to attribute differences in cost levels that are due to these factors to cost inefficiency. As a result, pricing policies in regulation that are conducted on the basis of comparative efficiency analysis would need to adjust for factors that can not be determined by the company and still have an impact on cost.

Another factor that has significant effect on costs can also be the quality of the good or service supplied. Provided that some measure of quality can be quantified, regression analysis is able to take into account the impact of quality, by assuming a particular functional form for this variable and therefore calculate on this basis the company's efficiency.

However, OLS and COLS have a major potential drawback: residuals in the estimation reflect a combination of relative efficiency, measurement error in the dependent variable (cost), and statistical noise, rather than only inefficiency. As a result, the final point-estimates of “efficiency” should be discounted before using them in formulating price policies for regulatory purposes.

Another limitation of OLS and COLS is that they are both subject to theoretical objections. In particular, the OLS method does not calculate a cost frontier that corresponds to the theoretical notion of a cost function but a fitted “average” function that provides no direct quantitative information on cost inefficiency in the sample. This objection is met to some extent from the use of COLS.

However, within COLS the (vertical) downward shift of the cost function to come up with a frontier implies that efficient and inefficient companies play the same role in the determination of the frontier. This is open to the objection that the most efficient companies should have a larger impact upon the shape of the frontier.

Finally, regression analysis is vulnerable to statistical problems including

- lack of “degrees of freedom”, which refer to the difference between the number of available observations and the number of potential explanatory factors. If there are too few degrees of freedom, this can lead to the problem of having too little information upon which to estimate the model;

- multicollinearity, which is a term that is used to describe a situation where the explanatory factors of the model are highly correlated. In this case, it could be extremely difficult to disentangle the individual influences of the explanatory factors and therefore the inferential procedure would be problematic;
Simple Regression Analysis

- the residual is not statistically independent of the explanatory factors (e.g. company size is correlated with efficiency), in which case the least squares estimators are biased upward or downward depending on the sign of the correlation;

- explanatory factors are measured with error, which results in least squares estimators being biased and inconsistent;\(^5\)

- omitted variables, in which case the least squares estimators are most likely to be biased except from the occasion on which the omitted variable is uncorrelated with the included explanatory variables; and

- residuals are not normally distributed and thereby standard inferential procedure becomes invalid.

Some of these problems can be intensified when the sample is relatively small, which can give rise to chance correlations amongst the explanatory variables or even between the explanatory variables and the residual.

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\(^5\) An estimator is said to be consistent when in very large samples its sampling distribution collapses onto a value equal to the parameter being estimated.
3 STOCHASTIC FRONTIER ANALYSIS

3.1 Introduction

The parametric approach of benchmarking also includes the stochastic frontier method. SFA differs from simple regression analysis in many respects. For example, whereas simple regression uses ordinary least squares to find the best fit of the average cost function, SFA uses mainly what are called “maximum likelihood” estimation techniques to estimate the frontier function in a given sample.

In addition, SFA separates error components from inefficiency components. In particular, it requires separate assumptions to be made as to the distributions of the “inefficiency” and “error” components, potentially leading to more accurate measures of relative efficiency.

3.2 Stochastic Frontier Analysis

SFA uses available data in order to estimate the cost function of a relatively efficient firm — known as the “frontier”. This function is assumed to be common for all firms and is used to obtain measures of inefficiency.

**Box 2 – The stochastic frontier model**

The stochastic frontier model can be written as follows:

\[ c_i = f(y_i; \beta) + w_i \]

\[ w_i = v_i + u_i \]

where \( f(y_i; \beta) \) represents the cost frontier, \( w_i \) is the total observed residual, \( v_i \) represents statistical noise and \( u_i \) is the inefficiency term. Since statistical noise can go in either direction, \( v_i \) has a mean value at zero, while \( u_i \) takes only non-negative values (actual cost \( c_i \) can never be lower than the frontier cost in the absence of data errors).

Estimation of the stochastic frontier model is usually implemented in two steps.

In the first step, a particular functional form for the relationship between cost and outputs and a functional form for the probability distribution of the efficiency term are assumed. By estimating the slope parameters \( \beta \), we obtain estimates for the frontier.

In the second step, we subtract actual cost from predicted cost (i.e. \( c - f(y; \beta) \)) and decompose the remaining residual \( w \) into a data error component \( v \) and an inefficiency component \( u \) for each company. The method is illustrated in figure 3.
As can be seen, for observations that lie above the frontier, the gap between each observation and the frontier is only partially attributed to inefficiency. The remainder of the gap is viewed as error in the measurement of the performance of each firm. For observations that lie below the frontier, the noise residual (v) is larger than the gap between the observation and the frontier in order to allow for some inefficiency. This implies that normally none of the firms will appear to be 100 per cent efficient.

### 3.3 Strengths and Weaknesses

SFA recognises the presence of errors and aims in principle to separate these error components from the measures of inefficiency. In practice, this effort is not always successful as, typically, the estimated inefficiency component represents a small fraction of the overall residual variation.

This practical nuance may cause many problems in the analysis. For example, it can make SFA vulnerable to outliers — that is, to observations that lie well above or below the main cluster of points.

The presence of outliers (that is, the presence of large residual variation) in the sample can cause the stochastic frontier model to perceive that there is too much noise in the data and therefore may find little or no inefficiency in the sample, even in cases where there is some. As a result, all companies may appear to be almost 100 per cent efficient. In this way, the main potential advantage of SFA of decomposing the residual into noise and inefficiency has turned to be a great disadvantage as it fails to differentiate between companies’ efficiency.

The problem of outliers is not a mere theoretical preconception as it may well occur in sectors where there are large discrepancies in the size of the comparators. In the UK water industry, for example, the difference in the size of Thames Water and Cambridge Water is huge. Putting both
companies in the same sample could well distort the final efficiency measures. To remedy this problem one could take these outliers out of the analysis and proceed without them. However, any such choice would be inherently arbitrary and difficult to be made.

There are also cases in which the stochastic frontier model ceases to have the role it intended to have. Sometimes, SFA can suggest that the noise residual has been drawn from a distribution with a very small variance. Consequently, deviations from the frontier are almost entirely due to the residual supposed to measure inefficiency. In these cases SFA collapses to a deterministic form, with the result that the frontier “envelopes” the observations from below, resulting in one at least company estimated to be 100 per cent efficient.

Another possibility is that the stochastic frontier model may detect little or no inefficiency because it suggests that the distribution of the residuals has the “wrong” skew.6 In these cases SFA collapses to simple OLS estimation. In a cost frontier model, “wrong” skew means that the residuals have no significant positive skew. This arises from the fact that the expected value of \( v \) is zero while \( u \) is always non-negative and therefore the observed residual \( (v + u) \) should be positively skewed in the presence of inefficiency.

SFA has the advantage – compared to non-parametric techniques, such as DEA (discussed below) – that it can provide some statistical inference as to the functional form of the frontier and the significance of individual explanatory factors upon the shape of the frontier. However, it can also be vulnerable to statistical problems of the nature discussed at the end of section 2.3. In addition, since the method uses maximum likelihood estimation, there is no guarantee that the final estimators will hold any desirable statistical properties (unbiasedness, efficiency, consistency) in small samples. Unfortunately, it is difficult to define a clear-cut sample size below which inferences become problematic as this will ultimately depend on the quality and nature of the data, the number of explanatory variables and the estimation procedure being followed.7

Finally, SFA is also subject to theoretical objections. In particular, the stochastic frontier model is an attempt to describe the true world within a sample of comparators by recognising the presence of both statistical errors and inefficiency in the data. To cope with this, it makes an assumption as to the functional form of the inefficiency effect. The most commonly used distributions are the half-normal and the exponential distribution. These distributions implicitly assume that there is a large number of relatively efficient firms and only few firms in the industry are relatively inefficient. In this way the shape of the frontier is almost equally affected by all data observations. In practice, however, most of the firms might be relatively inefficient. In this case, both distributions would be inappropriate, as they would attribute equal importance on efficient and inefficient companies in shaping the frontier. This has led to the development of more general — but also more complicated — distributions, such as the truncated-normal and the gamma distributions, for which the algebraic analysis is more complex but still practicable.

6 A distribution has zero skewness if it is symmetrical about its mean, in which case the mean, median and mode are equal. A distribution is positively (negatively) skewed if the right (left) tail is longer.
7 Note, however, that in empirical applications samples of size under 20 are usually considered to be small. In this case it may be sensible to use international comparators in order to increase the sample.
The main criticism against using these distributions for decomposing the residual is that there is often no a priori theoretical justification for selecting any of these distributions. The estimates of inefficiency may be sensitive to these alternative specifications, although the degree of sensitivity can vary from case to case.

Furthermore, the issue of selecting between different distributions according to which one fits best the data is not trivial, as the likelihood function to be maximised in each case is often significantly different.
4 DATA ENVELOPMENT ANALYSIS

4.1 Introduction

Data Envelopment Analysis is a non-parametric approach and relies on mathematical programming, rather than econometric techniques. By this we mean mainly the resolution of a set of problems via the maximisation/minimisation of a given objective subject to some constraints. DEA can perhaps be best explained by first introducing what are called “index numbers”.

4.2 Index Numbers

A simple method to estimate efficiency non-parametrically across firms would be to construct a simple index of relative performance such as the following:

\[
\text{efficiency score} = \frac{\beta_1 y_1 + \beta_2 y_2 + \ldots + \beta_k y_k}{\text{cost}}
\]

where \( y_1, y_2, y_k \) are different outputs and \( \beta_1, \beta_2, \beta_k \) are the weights attached to these outputs.

There are two main problems with simple indices of this kind. First, they implicitly assume a linear relationship between cost and outputs, that is, they assume constant returns to scale. This may be quite restrictive in the short-run.

Second, they restrict the weights attached to inputs/outputs to take the same value for all firms. This is often a disadvantage because in fact these weights may legitimately differ between firms, either because of different circumstances or because of variations in the importance attached to different inputs/outputs. As a result, simple index numbers tend to be rather uninformative.

4.3 Data Envelopment Analysis

A method within the non-parametric domain that avoids these restrictions is Data Envelopment Analysis (DEA). DEA uses mathematical linear programming techniques in order to find the set of weights for each firm \( \beta_1, \beta_2, \beta_k \) that maximises its efficiency score, subject to the constraint that none of the firms has an efficiency score greater than 100 per cent at those weights.

The purest version of DEA would start with the efficiency index used in equation (1). It would then allow the weights of this index to vary for each firm in such a way that each individual firm’s performance compares in the most favourable way with the remaining firms. The model would reject the solution for a particular firm if the set of weights that maximises its relative performance generate scores greater than 100 per cent for any other firm.

In this way, DEA builds up an “envelope” of observations that are most efficient at each set of weights. A firm can be shown to be inefficient if is less efficient than another firm at the set of

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8 Constant returns to scale exist when doubling all outputs leads to the doubling of cost. When doubling all outputs results in less than twice the cost, the cost function exhibits increasing returns to scale. Finally, decreasing returns exist when doubling all outputs leads to more than twice the cost.
weights that maximises its relative efficiency. For an inefficient firm at least one other firm will be more efficient with the target firm’s set of weights. These efficient firms are known as the peer group for the inefficient firm.

A DEA model not only allows the weights attached to each performance indicator to vary across firms, but is also able to accommodate non-linear relationships between cost and outputs — that is, variable returns to scale (VRS). In this respect, DEA may be viewed as an extension of simple index numbers.

The VRS DEA model is illustrated in figure 4, for the case of a single input.

As it is illustrated in the figure, DEA approximates the best-practice frontier by a deterministic “piece-wise” linear approximation based on the available sample, and subsequently attributes the gap between any observation and the frontier to inefficiency. Therefore, organisations C, E and F would be considered as efficient. By contrast, organisation D lies inside the frontier and is considered to be inefficient. Moreover, for organisation D the peer group consists of organisations C and E, and its target cost level is given by D’.

4.4 Strengths and Weaknesses

Comparing to COLS and SFA, DEA has the advantage that it does not need to employ an assumption for the functional form of the frontier other than the minimum piecewise and linear condition. As a result, there is no danger of mis-specifying the frontier in this way. On the other hand, this lack of parameterisation is also a disadvantage, as it is very difficult to use the data to

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guide mode choice – for example, there is no proper definition of goodness of fit that would enable comparison of different models during the modelling procedure.

DEA is computationally less intensive than SFA (at least in its basic form) and for this reason the method has been more widely used, especially in operations research.

Moreover, compared to regression analysis and partly SFA, DEA has the advantage that it takes into account only the most efficient companies in shaping the frontier.

DEA adopts the weights for each firm that maximise each firm’s relative performance. One of the main shortcomings of DEA for relative efficiency analysis therefore is that rather too many of the firms may appear to be efficient, even if this is not truly the case. This problem can be intensified when the sample of comparators is small and the number of outputs large. This is because the dimensions in which a particular firm can be unique increase and therefore its potential peer group is narrower.

In practice, weight restrictions can be used to ensure that neither exceptionally high weights are placed on a number of relatively unimportant outputs, nor that a relatively important output plays only a minor role in the determination of the efficiency measure. However, there is no single way of selecting weight restrictions and each of them has its own limitations when being applied.9

A major drawback of DEA is that it attributes all deviations from the frontier to inefficiency. Yet, as with regression analysis, deviations from the frontier may be due to a number of factors other than inefficiency such as omitted cost drivers and measurement errors. These factors are not testable. As a result, interpreting DEA scores as measures of efficiency requires a high degree of “blind” faith in the model. In fact, the most that one can argue objectively is that DEA scores show the amount of allowable costs that the model has justified. The remaining gap between the observation and the frontier remains unexplained.

The deterministic nature of DEA can cause significant problems in the measurement of efficiency when there are outliers in the industry because the method envelops the outermost observations without asking whether these observations are genuine or the result of an error. Even a single outlier can result in finding huge inefficiencies for most comparators without this being necessarily true. To remedy this problem and find “sensible” scores of inefficiency, one could take these outliers out of the analysis and proceed without them, although there is no clear way of deciding which companies should be regarded as outliers and which not.

Therefore, we can see that outliers can cause problems in both SFA and DEA but for completely different reasons: while SFA can fail to find any inefficiency at all, DEA is likely to find too much inefficiency in the sample.

5 CONCLUDING REMARKS

5.1 Choosing between Comparative Efficiency Measurement Techniques

Comparative efficiency analysis is a useful tool in benchmarking. However, choosing between different comparative efficiency methods is not trivial, especially because the techniques are fundamentally different from each other and therefore it is quite likely that they yield different results.

Some researchers have undertaken large-scale simulations to examine the relative performance of the stochastic frontier models vis-à-vis DEA.\textsuperscript{10} It has been shown that if the employed functional form for the frontier is close to the “true” underlying technology, stochastic frontier models outperform DEA. As the mis-specification of the functional form of the frontier becomes more serious or as the degree of correlation of the regressors with inefficiency increases, DEA becomes more appealing. Furthermore, in cases where noise was important in the data, DEA’s performance was, as expected, substantially lower than the performance of stochastic frontier models.

Consequently, we see that in cases where

- random influences and statistical noise are perceived to influence heavily the data;
- there is confidence that the functional form of the frontier has been well specified;
- omitted variables may influence the results; and
- hypothesis testing is important;

the SFA approach is likely to be more appropriate.

On the other hand, DEA may be preferable in cases where

- the regressors are highly correlated;
- random influences are less of an issue;
- it is difficult to justify a particular functional form for the inefficiency component; and
- behavioural assumptions such as cost minimisation are difficult to justify.

In many practical cases, however, the sample might be far too small to make inferences upon the choice of technique with certainty. As a result, the best approach would often be the use of

Concluding Remarks

different techniques in tandem. This implies that one can opt for a spread of relative efficiency estimates rather than a single ranking of relative efficiency based on a single method.

5.2 Comparing and Interpreting the Final Efficiency Measures

One of the main conclusions of this paper is that in fact no method is free from criticism. All methods face their own problems both on the theoretical and the practical side. This implies that the final efficiency estimates should not be interpreted as being definitive measures of inefficiency. By contrast, a range of efficiency scores may be developed and act as a signalling device rather than as a conclusive statement.

The final efficiency scores are most likely to be sensitive to the methodology chosen. In particular, the mean of the SFA estimated efficiencies will always be larger than that of COLS or DEA efficiencies. The opposite is true with respect of the variance of the efficiency scores. This is happening for two reasons.

First, because the non-stochastic methods, such as DEA and COLS, treat residuals as if they were measures of relative efficiency. In practice, part of the residual is likely to reflect measurement errors and statistical noise. As a result, companies that lie away from the frontier achieve a lower score than they would have achieved, had some portion of the residual been attributed to random noise.

On other hand, the stochastic approach (SFA) cannot always separate successfully error components from measures of inefficiency as the latter typically represent some fraction of the overall residual variation. This can lead to an upward bias in the efficiency scores, a phenomenon that is more common when the efficiency component is assumed to follow the exponential distribution.

This fact indicates that an objective efficiency estimate for each company is likely to lie somewhere between the estimates of efficiency of the stochastic and non-stochastic methods. Some researchers suggest the use of geometric means of the scores of the preferred models to reduce the possible bias in individual models.11

Deciding upon the preferred models within each approach is not a trivial thing, especially in DEA where there are no rules that can deal satisfactorily with what is termed in econometrics as the model specification problem. For this reason, some sensitive analysis may be desirable to crosscheck the validity of the final findings.

The validity of the results can be divided into internal and external validity. The former reflects the possibility that the models used can change the results significantly. A test of internal validity can be the comparison of the results obtained using genuinely competing models with different selections of inputs (where applicable) and outputs.

External validity raises the question of whether the results are applicable more generally. A test of external validity can be the consistency of the efficiency scores over time. In particular, while it is natural to assume that there will be some changes in efficiency through time, it is unlikely that these changes would be very large between two years that the final results would be radically different.

5.3 Moving Forward

Comparative efficiency analysis has been used extensively by regulators and has played an important part of the regulatory process.

In some industries the easy efficiency gains have already been extracted and most of the companies are expected to converge to the efficiency frontier within a pre-set period of time. This is likely to boost the value of establishing the potential for shifting the frontier for the companies individually or the industry as a whole. As a result, techniques that make use of company-specific data over a period of time (such as panel data analysis) rather than a single year will become increasingly important in the future.

Such a situation will raise the need for regulators to develop a common and, most important, time-consistent accounting framework within each sector. Differences in the capitalisation policies used in reporting accounting data over time, or different asset management policies, would unduly distort the possibility of assessing shifts in the efficiency frontier.

This can also increase the significance of international benchmarking, which may bring with it increased emphasis on the importance of understanding and allowing for environmental factors, and of course introduces the issue of the role of exchange rates in efficiency analysis.
APPENDIX 1: RECENT DEVELOPMENTS

A1.1 Comparative Efficiency Analysis with Panel Data

The models considered in the main text are cross-sectional in the sense that we observe each company at a single point in time. In a panel data set we observe each company not only once but over a period of time and thereby our ability to make statistical inferences increases. Panel data models tend to be less susceptible to multicollinearity and degrees of freedom problems.

Furthermore, if assumptions about the functional form of the distribution of the inefficiency effects are difficult to justify, and the functional form of the relationship between cost and outputs requires a lot of data for estimation to proceed, it is desirable to use panel data analysis. In particular, panel data analysis avoids making strong distributional assumptions about the inefficiency effects (i.e. half-normal, exponential and so on). By contrast, these effects are usually assumed to be either fixed or random — and hence the fixed and random effect models. This means that in the former model the firm-specific inefficiency effects are treated as fixed, while in the latter model the firm-specific inefficiency effects are treated as realisations of some random process (common for all companies).

Box 3 – The stochastic frontier model with panel data

The panel data frontier model may be written as follows:

\[ c_{it} = \beta_{0i} + f(y_{it};\beta) + v_{it} \]

where \( \beta_{0i} \) is a variant for the inefficiency term and \( t \) is a subscript for time.

As we can see, the main benefit of panel data models is that the inefficiency term — given by \( \beta_{0i} \) — becomes a parameter to be estimated within the model rather than a component to be decomposed from the residual.

Another advantage of panel data analysis is that the efficiency estimates that it provides are statistically consistent. By contrast, cross-sectional SFA provides estimates that although unbiased they are inconsistent. The inconsistency of the estimator of \( u_i \) is quite problematic since the purpose of the analysis to begin with is the estimation of efficiency. Unfortunately, this appears to be the best that can be achieved with cross-sectional data. The problem arises not because estimates of \( u_i \) converge to a non-true value but rather because the variance of the distribution of \( u_i \) conditioned on \( w_i \) remains non-zero even in large samples.

The success of panel data analysis depends crucially on the validity of the functional form employed for the frontier. If the assumed form is wrong, the estimates will be biased, as is the case for the cross-sectional models. However, the relative advantage of panel data models is that they include more information and therefore tests regarding the functional form of the frontier may be more reliable.
A specific issue that arises in panel data is that of modelling the time aspect of inefficiency. In traditional panel data models efficiency is assumed to remain unchanged through time. This assumption may be difficult to justify particularly when we observe the same firms over a long period of time. For example, it is natural to think that efficiency will improve over time as new working practices are developed, and differences in efficiency may narrow if firms can learn from each other’s practices. Panel data analysis enables one not only to check whether a company’s efficiency is improving over time relative to the frontier, but also whether the frontier itself is shifting.

**Box 4 – Panel data analysis with time-varying inefficiency**

Assuming that the frontier function is expressed in translog form, the model may be written as follows:

\[ c_{it} = \beta_0 + \sum_k \beta_{ky_{ikt}} + \beta_t + \frac{1}{2} \left( \sum_k \sum_q \beta_{kq} y_{ikt} y_{qit} + \beta_{tt} t^2 \right) + \sum_k \beta_{kt} y_{ikt} t + v_{it} + u_{it} \]

where \( y \) and \( x \) are the respective output and inputs measured in natural logarithms, \( t \) denotes time, \( k \) and \( q \) denote input indices, \( v \) is the usual statistical error and \( u \) is the non-negative inefficiency term.

The equation above differs from the equation analysed in Box 3 in two respects. First the inefficiency term \( u \) is allowed to vary through time. This is important because it is natural to think that differences in efficiency between firms may narrow over time as firms learn from each other’s practices. However, the way inefficiency varies cannot be unconstrained since otherwise the model will not be identified — that is, the number of parameters to be estimated will exceed the number of available observations.

Lee and Schmidt (1993) have proposed the following formulation for inefficiency: \( u_{it} = f(t)u_i \), where the function \( f(t) \) is specified as a set of dummy variables for time.\(^{12}\) This is not of course the only way to model time-varying inefficiency and unfortunately it is not always clear which method to choose.

The second way that the equation above differs from the equation in Box 3 is the inclusion of \( t \) as a regressor. This allows one to make useful inferences about the change in the frontier as the industry-wide technology improves over time. For example, the efficiency frontier will remain unchanged if \( \beta_t = \beta_{tt} = \beta_{kt} = 0 \) or it will be “neutral” with respect to the inputs if \( \beta_{kt} = 0 \) for all \( k \).

An important issue that arises with panel data is that of stationarity. In simple terms, a variable is non-stationary if its mean, variance or covariance with another variable changes over time. It has

been shown recently that when the issue of stationarity is not taken into account in comparative efficiency analysis with panel data, efficiency measurement may lead to biased results.\footnote{Tsionas, E.G. and Christopoulos, D.K (2001) “Efficiency measurement with non-stationary variables: An application of panel cointegration techniques.” Economics Bulletin, Vol. 3, No 14, pp. 1-7.}

### A1.2 Stochastic Data Envelopment Analysis

Stochastic DEA (SDEA) is the result of new efforts to accommodate stochastic influences on the DEA frontier and in this way to face the relevant criticism attributed to DEA — that is, the fact that DEA essentially assumes that all deviations from the frontier are due to inefficiency.

In essence, SDEA solves the problem by maximising the efficiency score of each company but allowing a certain probability that the normal DEA constraints will not be satisfied.

This can be seen more clearly in figure 6 below where the SDEA method associates the observations with the stochastic influences of the model and therefore the frontier passes through the “realisation” of companies created by making an assumption that the DEA constraints will be satisfied only with some probability. As a result, different realisations occur for each company according to the probability we attach to the satisfaction of the constraints. The introduction of this probabilistic element in the model has the effect of moving the SDEA frontier closer to the bulk of the companies. The smaller the probability that a single constraint will be satisfied for a company, the smaller will be the distance between this company and the SDEA frontier. Consequently, the SDEA score will never be lower than the DEA efficiency score.

Note that because of the noise that is assumed in the data, some companies will lie above the frontier and therefore they will (using the DEA jargon) be characterised as “super-efficient.”
SDEA has not been widely used and only a limited amount of research has been done in applying it in empirical studies. The main problem is that the introduction of the probabilistic element in the model is quite arbitrary and the results can be very sensitive to this assumption.